VaR Without Correlations for Nonlinear Portfolios

By Giovanni Barone-Adesi

University of Alberta, Canada

&

City University Business School, London

Kostas Giannopoulos

University of Westminster, London

Les Vosper

The London Clearing House Ltd

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Abstract

We propose filtering historical simulation by GARCH processes to model the future distribution of assets and swap values. Options' price changes are computed by full re-evaluation on the changing prices of underlying assets. Our methodology takes implicitly into account assets' correlations without restricting their values over time or computing them explicitly. VaR values for nonlinear portfolios are obtained without linearising them. Historical simulation assigns equal probability to past returns, neglecting current market conditions.

1 INTRODUCTION

Current methods of evaluating the risk of portfolios of nonlinear securities are unsatisfactory. Delta-gamma hedging becomes unstable for large asset price changes or for options at the money with short maturities. Monte-Carlo simulations assume a particular distributional form, imposing the structure of risk that they were supposed to investigate. Moreover, they often use factorisation techniques that are sensitive to the ordering of the data.

To overcome the above limitations we propose to adapt the methodology of Barone-Adesi, Bourgoin & Giannopoulos (1997). We propose to model changes in asset prices to depend on current asset volatilities. Asset volatilities are simulated to depend on the most recently sampled portfolio returns. Our simulation is based on the combination of GARCH modelling (parametric) and historical portfolio returns (non-parametric). Historical returns are adapted to current market conditions by scaling them by the ratio of current over past conditional volatility.

The scaled returns are the basis of our simulation. To simulate a pathway of returns for each of a number of different assets over next 10 days we select randomly 10 past sets or "strips" of returns, each return in a strip corresponding to an asset's price change which occurred on a day in the past. Thus each strip of returns represents a sample of the co-movements between asset prices. We then iteratively construct the daily volatilities for each asset that each these strips of returns imply according to our GARCH model.

We use these volatilities to scale each of our sampled returns. The resulting returns therefore reflect current market conditions rather than historical ones.

Many VaR models assume that asset returns follow a normal distribution. Normality simplifies VaR calculation because all percentiles are assumed to be multipliers of the standard deviation. A number of studies, however, have found that the empirical distributions of returns are non-normal i.e. they have fat-tails and non-zero skewness. In that case assuming normality in calculating VaR leads to the under-prediction of uncommon (but possible) losses.

The core of our methodology is the historical returns of the data. The “raw” returns, however, are unsuitable for historical simulation because they do not fulfil the properties[1] necessary for unbiased results.

Among others Mandelbrot (1963) found that most financial series contain volatility clusters. In VaR analysis, volatility clusters imply that the probability of a specific loss being incurred is not the same on each day. During days of higher volatility we will expect larger than usual losses.

2 SIMULATING A SINGLE PATHWAY

In our simulation we do not impose any theoretical distribution on the data. We use the empirical (historical) distribution of the return series. However, “raw” returns cannot be used in the simulation unless they are i.i.d. (independently and identically distributed). To render returns i.i.d. we need to remove any serial correlation and volatility clusters present in the dataset. Serial correlations can be removed by adding an MA term in the conditional mean equation. One possible way to remove volatility clusters is to write the conditional variance as an autoregressive heteroskedastic process. Hence, we model returns as a GARCH process (Bollerslev,1986) and consequently capture any volatility clusters[2]. When appropriate we insert a moving average (MA) term in the conditional

[1] For simulation, returns should be random numbers drawn from a stable distribution i.e. they should be identically and independently distributed (i.i.d.).

[2] The particular form of GARCH process used for a series was determined by statistical testing.

mean equation (1) to remove any serial dependency. Therefore, an ARMA-GARCH(1,1) model can be written as:

$$r_t \;=\; m\; r_{t-1} \;+\; \theta e_{t-1} \;+\; \hat{\imath}_t \qquad\qquad \hat{\imath}_t \sim \mathrm{N}(0, h_t) \tag{1}$$

$$h_t \;=\; \omega \;+\; \grave{a} e^2_{t-1} \;+\; \beta h_{t-1} \tag{2}$$

where m is the AR(1) term, θ is the MA term, ω is a constant and $\hat{\imath}_t$ the random residual. The GARCH(1,1) equation defines the volatility of $\hat{\imath}_t$ as a function of the constant ω plus two terms reflecting the contributions of the most recent surprise $\hat{\imath}_{t-1}$ and the last period's volatility h_{t-1}, respectively.

To bring returns close to a stable distribution we need to divide the residual, $\hat{e}_t$ by the corresponding daily volatility estimate, $\sqrt{\hat{h}_t}$. Thus, the standardised residual return is given as:

$$\hat{e}_t \;=\; \frac{\hat{e}_t}{\sqrt{\hat{h}_t}} \tag{3}$$

This makes the set of standardised residuals independently and identically distributed (i.i.d.) and therefore suitable for historical simulation.

As Barone-Adesi, Bourgoin and Giannopoulos (1998) have shown, historical standardised innovations can be drawn randomly (with replacement) and after being scaled with current volatility, may be used as innovations in the conditional mean (1) and variance (2) equations to generate pathways for future prices and variances respectively. This methodology stands as follows:

- we draw standardised residual returns as a random vector é of outcomes from a stationary distribution:

$$e^* = \{ e_i^* = \text{é}\}, \text{é} = \{ e_1, e_2,..., e_T\} \tag{4}$$

where i = 1,....,10 days.

- to get the innovation forecast (simulated) value for period t+1, z^*_{t+1}, we draw a random standardised residual return from the dataset é and scale it with the volatility of period[3] t+1 :

$$z^*_{t+1} = e^*_1 \,.\, \sqrt{\hat{h}_{t+1}} \tag{5}$$

- we begin simulation of the pathway of the asset's price from the currently known asset price, at period t. The simulated price p^*_{t+1} for t+1 is given as

$$p^*_{t+1} = p_t + p_t(\, m\; r_t + \theta\; z^*_t + z^*_{t+1}) \tag{6}$$

where z^* is estimated as in (5).

For i = 2, 3... the volatility is unknown and must be simulated from the randomly selected re-scaled residuals. In general $\sqrt{h^*_{t+i}}$, the (simulated) volatility estimate for period t+i, is obtained as:

$$\sqrt{h^*_{t+i}} = \sqrt{w + a(z^*_{t+i-1})^2 + b h^*_{t+i-1}} \qquad i \geq 2 \tag{7}$$

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[3] The variance of period t+1 it can be calculated at the end of period t as:

$h_{t+1} = \omega + \text{à} e_t^2 + \beta h_t$, in which $\hat{\imath}_t$ is the latest actual residual return in (1).

where z^* is estimated as in (5).

New elements e_t^* are drawn from the dataset é to form the simulated prices p_{t+i}^* as in (6).

The distribution of evolved prices at the end of the pathway (e.g. when i = 10) for the single asset is obtained by replicating the above procedure a large number of times e.g. 5000.

3 SIMULATING MULTIPLE PATHWAYS

To estimate risks for a portfolio of multiple assets we need to preserve the multivariate properties of asset returns; however, methodologies which use the correlations matrix of asset returns encounter various problems with this. The use of conditional multivariate econometric models which allow for correlations to change over time, is restricted to a few series at a time. The data inputs in a correlations matrix increase with the square of the number of assets in the portfolio: for large portfolios the number of pairwise correlations becomes unmanageable.

When estimating time-varying correlation coefficients independently from each other, there is no guarantee that the resulting matrix satisfies the multivariate properties of the data. In fact the resulting matrix may not be positive definite.

Additionally, the estimation of VaR from the correlations matrix requires knowledge of the probability distribution of each asset series. However, empirical distributions may not conform to any known distribution: often the empirical histograms are smoothed and forced to follow a known distribution convenient for the calculations. VaR measures which are based on arbitrary distributional assumptions may be unreliable; preliminary smoothing of data can cover up the non-normality of the data and VaR estimation, which is highly dependent on the good prediction of uncommon events, may be adversely affected from smoothing the data.

Finally, correlations measured from daily returns can be demonstrated to be unstable. Even their sign is ambiguous. Estimated correlations coefficients can be the subject of such great changes at any time, which even conditional models do not capture, that the successful forecast of portfolio losses may be seriously inhibited.

Our approach does not employ a correlation matrix. For a portfolio of multiple assets we extend our bootstrapping methodology[4] to simulate multiple pathways. We select a random date from the dataset, which will have an associated set of i.i.d. residual returns for each asset. This “strip” of i.i.d. residual returns, derived at a common date in the past, is one sample from which we begin modelling the co-movements between respective asset prices.

Thus for each asset for i = 1,...,10 days we have the sampled residuals denoted by subscripts 1, 2, 3,...., for the different assets.

Asset 1: $e_1^* = \{ e_{1,i}^* = \acute{e}_1 \}, \acute{e}_1 = \{ e_1, e_2, ..., e_T \}_1$ (8)

Asset 2: $e_2^* = \{ e_{2,i}^* = \acute{e}_2 \}, \acute{e}_2 = \{ e_1, e_2, ..., e_T \}_2$ (9)

Asset 3: $e_3^* = \{ e_{3,i}^* = \acute{e}_3 \}, \acute{e}_3 = \{ e_1, e_2, ..., e_T \}_3$ (10)

and so on for all the assets in the dataset.

From the dataset $\acute{e}$ of historical standardised innovations, for i = 1, a date is randomly drawn and hence the associated residuals e_1^*, e_2^*, e_3^* are selected. At i = 2 another date is drawn, with its corresponding residuals, and so on for i = 3, 4....etc. Thus pathways

[4] See Efron and Tibshirani (1993), “An Introduction to the Bootstrap”, in which the bootstrapping methodology is introduced and described in depth.

for variances and prices are constructed for each asset which reflect the co-movements between asset prices:

For i = 1 to 10:

Asset 1:
$$h^*_{1,t+i} = \omega_1 + \alpha_1(z^*_{1,t+i-1})^2 + \beta_1 h^*_{1,t+i-1} \tag{11}$$

$$p^*_{1,t+i} = p^*_{1,t+i-1} + p^*_{1,t+i-1}(m_1\, r_{1,t+i-1} + q_1\, z^*_{1,t+i-1} + z^*_{1,t+i}) \tag{12}$$

Asset 2:
$$h^*_{2,t+i} = \omega_2 + \alpha_2(z^*_{2,t+i-1})^2 + \beta_2 h^*_{2,t+i-1} \tag{13}$$

$$p^*_{2,t+i} = p^*_{2,t+i-1} + p^*_{2,t+i-1}(m_2\, r_{2,t+i-1} + q_2\, z^*_{2,t+i-1} + z^*_{2,t+i}) \tag{14}$$

Asset 3:
$$h^*_{3,t+i} = \omega_3 + \alpha_3(z^*_{3,t+i-1})^2 + \beta_3 h^*_{3,t+i-1} \tag{15}$$

$$p^*_{3,t+i} = p^*_{3,t+i-1} + p^*_{3,t+i-1}(m_3\, r_{3,t+i-1} + q_3\, z^*_{3,t+i-1} + z^*_{3,t+i}) \tag{16}$$

where z^* is estimated as in (5).

4 AN EMPIRICAL INVESTIGATION

We illustrate our methodology with a numerical example of a portfolio of three assets. In addition we use portfolios of futures and options traded on the various exchanges in London and portfolios of “plain vanilla” interest rate swaps, to demonstrate some results of our application.

4.1 Description of Dataset for London Exchanges

We use two years of historical daily prices for futures contracts traded on the London exchanges. From these we form logarithmic returns r_t for each of the futures price series:

$$r_t \quad = \quad \ln\left(\frac{p_t}{p_{t-1}}\right) \tag{17}$$

where p_t = daily prices for that series.

These returns form the basis of our dataset; we form the i.i.d. residual returns from them as described in equations (1) to (3).

For swaps evaluations we employ two years of historical daily zero coupon interest rates, obtained from Reuters, for 1 day, 30 days, 90 days, 180 days, 360 days, 2 years, 3 years, 4 years, 5 years, 7 and 10 years, in four currencies: US Dollar, Sterling, Japanese Yen and Deutschemarks.

From these we form logarithmic returns as in equation (17); i.i.d. residual returns are obtained from these, again as in equations (1) to (3).

4.2 Example

Consider a portfolio of 3 assets (e.g. commodity futures), with net lots 2, -5 and 10 respectively. The current close of business is 7/2/97 and we wish to estimate the distribution of portfolio values for 2 days ahead. GARCH forecast estimates of asset annual volatilities for one day ahead (i.e. 8/2/97) are 10%, 15% and 8% respectively[5]. Asset prices are currently \$1700, \$2000 and \$1900 per tonne of asset; the asset prices are quoted at 25 tonnes per lot. Conditional mean returns are assumed for simplicity to be

[5] These volatilities are annualised; we need to divide them by $\sqrt{252}$ to make them into daily values.

zero at all times.[6] The database of historical, normalised asset returns is illustrated in Table 1 below:[7]

Table 1

	Asset 1	Asset 2	Asset 3	...	Asset n
.	.	.	.	...	.
.	.	.	.	...	.
1-Sep-95	-1.38141	-1.01865	0.25736	...	.
4-Sep-95	-2.10091	0.182623	0.220097	...	.
5-Sep-95	1.205624	0.036487	-0.88315	...	.
6-Sep-95	-0.11472	0.908106	-0.51859	...	.
7-Sep-95	-0.53701	-0.36231	0.185504	...	.
8-Sep-95	-1.04343	-0.32714	-0.93079	...	.
11-Sep-95	-2.95359	0.072785	-0.78824	...	.
.	.	.	.	...	.
.	.	.	.	...	.
21-May-96	-0.80552	-0.54346	0.278256	...	.
.	.	.	.	...	.
.	.	.	.	...	.
13-Nov-96	-2.81372	0.244584	0.294773	...	.
.	.	.	.	...	.
.	.	.	.	...	.
7-Feb-97	0.824807	1.476361	2.836008	...	.

We make the first random selection of a date: referring to the table of normalised values, suppose this to be on 21 May 96[8]. The row of residual returns is (-0.80552, -0.5346, 0.278256,.....) for each of the assets.

[6] For some futures prices this is not an altogether unreasonable assumption; it means we can ignore any AR terms. MA terms are also ignored; much of our data in fact had zero MA terms. Thus we set q and m to zero.

[7] Table 1 is an extract, for illustrative purposes, of normalised residual returns based on closing prices for three commodity futures over a two year period. We can have as many columns of residual returns as there are assets, or as in the case of swaps in a given currency, a set of columns of interest rate residual returns e.g. from 1 day to 10 years per currency, from which swap evaluations may be performed.

[8] As the random sampling is with replacement, we may draw the same date more than once during the simulation process.

Each residual is re-scaled by the forecast *daily* volatility $\sqrt{h^*_{t+1}}$ of the corresponding asset:

Asset 1: $z^*_{1,t+1}$ = -0.80552 x $\frac{0.1}{\sqrt{252}}$ = -0.00507

Asset 2: $z^*_{2,t+1}$ = -0.5346 x $\frac{0.15}{\sqrt{252}}$ = -0.00514

Asset 3: $z^*_{3,t+1}$ = 0.278256 x $\frac{0.08}{\sqrt{252}}$ = 0.001402

We now apply each re-scaled return to the current asset price at the close of business. This will give a forecast of each asset's price for one day ahead, based on how prices moved together on 21 May sample.

Asset 1: $p^*_{1,t+1}$ = \$1700 + (\$1700 x -0.00507) = \$1691.38

Asset 2: $p^*_{2,t+1}$ = \$2000 + (\$2000 x -0.00514) = \$1989.72

Asset 3: $p^*_{3,t+1}$ = \$1900 + (\$1900 x 0.001402) = \$1902.66

Using the re-scaled residual returns i.e. from 21 May, in the GARCH variance equations for each asset, we next compute the variance forecasts for 2 days ahead:

Asset 1: $h^*_{1,t+2} = \omega_1 + \alpha_1(z^*_{1,t+1})^2 + \beta_1 h^*_{1,t+1}$

where $z^*_{1,t+1}$ = -0.00507 and variance $h^*_{1,t+1} = \left(\frac{0.1}{\sqrt{252}}\right)^2$. Assuming the coefficients ω_1, α_1, and β_1 for this asset as 0.00001552, 0.1 and 0.642 respectively, we have for the standard deviations:

$$\sqrt{h^*_{1,t+2}} \quad = \quad \sqrt{w_1 + a_1(-0.00507)^2 + b_1(0.0063)^2} \;=\; 0.0066$$

Similarly we get

Asset 2: $\sqrt{h^*_{2,t+2}}$ = 0.0092

Asset 3: $\sqrt{h^*_{3,t+2}}$ = 0.0054

The next step is obtain the 2 day ahead price forecasts: we randomly sample a second date in the database: suppose this to be 13 November 1996. The respective standard-ised returns this time are -2.81372, 0.244584 and 0.294773. These are re-scaled by the 2 day ahead volatility forecasts computed in the previous step:

Asset 1: $z^*_{1,t+2}$ = 0.0066 x -2.81372 = -0.0186

Asset 2: $z^*_{2,t+2}$ = 0.0092 x 0.244584 = 0.00225

Asset 3: $z^*_{3,t+2}$ = 0.0054 x 0.294773 = 0.00159

We then apply these returns to the 1 day forecast prices to obtain the 2 day forecast prices:

Asset 1: $p^*_{1,t+2}$ = \$1691.38 + (\$1691.38 x -0.0186) = \$1659.92

Asset 2: $p^*_{2,t+2}$ = \$1989.72 + (\$1989.72 x 0.00225) = \$1994.20

Asset 3: $p^*_{3,t+2}$ = \$1902.66 + (\$1902.66 x 0.00159) = \$1905.69

The 2 day ahead price pathways starting from the close of business are:

For i = (1, 2):

Asset 1: $p^*_{1,\{t,t+1,t+2\}}$ = \$1700, \$1691.38, \$1659.92

Asset 2: $p^*_{2,\{t,t+1,t+2\}}$ = \$2000, \$1989.72, \$1994.20

Asset 3: $p^*_{3,\{t,t+1,t+2\}}$ = \$1900, \$1902.66, \$1905.69

for the first simulation. If we wish to increase the length of paths e.g. to 10 days ahead, we continue the procedure for the eight further steps.

4.3 Aggregating Asset Pathways to Obtain Portfolio Pathways

For the first simulation we select the asset pathways which correspond to the contracts in the portfolio. These are the vectors

$$\mathbf{p}_{1,\{t,\,t+1,\ldots,\,t+i\}}\ ,\ \mathbf{p}_{2,\{t,\,t+1,\ldots,\,t+i\}}\ ,\ \mathbf{p}_{3,\{t,\,t+1,\ldots,\,t+i\}}\ ,\ldots\ldots,\mathbf{p}_{n,\{t,\,t+1,\ldots,\,t+i\}} \tag{18}$$

for n assets and a time horizon of i days. If the positions i.e. net lots of contracts in the portfolio are represented by the scalars w_1, w_2, w_3,....,w_n then the position-weighted pathways are the vectors

$$w_1\mathbf{p}_{1,\{t,\,t+1,\ldots,\,t+i\}}\ ,\ w_2\mathbf{p}_{2,\{t,\,t+1,\ldots,\,t+i\}}\ ,\ w_3\mathbf{p}_{3,\{t,\,t+1,\ldots,\,t+i\}}\ ,\ldots\ldots,\ w_n\mathbf{p}_{n,\{t,\,t+1,\ldots,\,t+i\}} \tag{19}$$

The vectors of pathways are added to form the portfolio path $\boldsymbol{\pi}_{t,\,t+1,\ldots,t+i}$

$$\boldsymbol{\pi}_{t,\,t+1,\ldots,t+i} = w_1\mathbf{p}_{1,\{t,\,t+1,\ldots,\,t+i\}} + w_2\mathbf{p}_{2,\{t,\,t+1,\ldots,\,t+i\}} + w_3\mathbf{p}_{3,\{t,\,t+1,\ldots,\,t+i\}} +,\ldots,+ w_n\mathbf{p}_{n,\{t,\,t+1,\ldots,\,t+i\}} \tag{20}$$

Continuing our example to illustrate:

From the closing prices and for i = (1, 2) the paths are

$$\text{Asset 1: } w_1\mathbf{p}^*_{1,\{t,t+1,t+2\}} = [2(\$1700),\ 2(\$1691.38),\ 2(\$1659.92)]$$
$$= [\$3400,\ \$3382.76,\ \$3319.84]$$

$$\text{Asset 2: } w_2\mathbf{p}^*_{2,\{t,t+1,t+2\}} = [-5(\$2000),\ -5(\$1989.72),\ -5(\$1994.20)]$$
$$= [-\$10{,}000,\ -\$9948.60,\ -\$9971]$$

$$\text{Asset 3: } w_3\mathbf{p}^*_{3,\{t,t+1,t+2\}} = [10(\$1900),\ 10(\$1902.66),\ 10(\$1905.69)]$$
$$= [\$19{,}000,\ \$19{,}026.60,\ \$19{,}056.90]$$

The portfolio path based on prices per tonne is

$$\boldsymbol{\pi}_{t,\ t+1,\ t+2} = w_1\mathbf{p}^*_{1,\{t,t+1,t+2\}} + w_2\mathbf{p}^*_{2,\{t,t+1,t+2\}} + w_3\mathbf{p}^*_{3,\{t,t+1,t+2\}}$$

$$\begin{aligned} = &\quad [\$3400,\ \$3382.76,\ \$3319.84] \\ + &\quad [-\$10{,}000,\ -\$9948.60,\ -\$9971] \\ + &\quad [\$19{,}000,\ \$19{,}026.60,\ \$19{,}056.90] \end{aligned}$$

$$= [\$12{,}400,\ \$12{,}460.76,\ \$12{,}405.44]$$

Multiplying by the lot size of 25 tonnes per lot gives the portfolio value pathway for the first simulation:

$$\boldsymbol{\pi}_{t,\ t+1,\ t+2} = [\$310{,}000,\ \$311{,}519,\ \$310{,}144]$$

The change in the portfolio's value after 2 days, from its closing value is \$310,144 - \$310,000 = \$144 which for this (first) simulation is a gain in value.

5 DISTRIBUTIONS

To obtain a distribution of portfolio values, we replicate the whole of the above procedure 5000 times, obtaining 5000 simulated portfolio values for a given time horizon. From these we construct the probability distribution function for the portfolio.

Figures 1 and 2 illustrate examples of distributions of price pathways, in this case for the LIFFE German Bund financial futures contract (the near month was used), for 5000 simulations, one day and ten days ahead respectively. We note in particular how far from the normal distribution, is the one-day ahead histogram of prices:

Figure 1

The 1-day ahead distribution of German Bund Futures Prices over 5000 Simulations

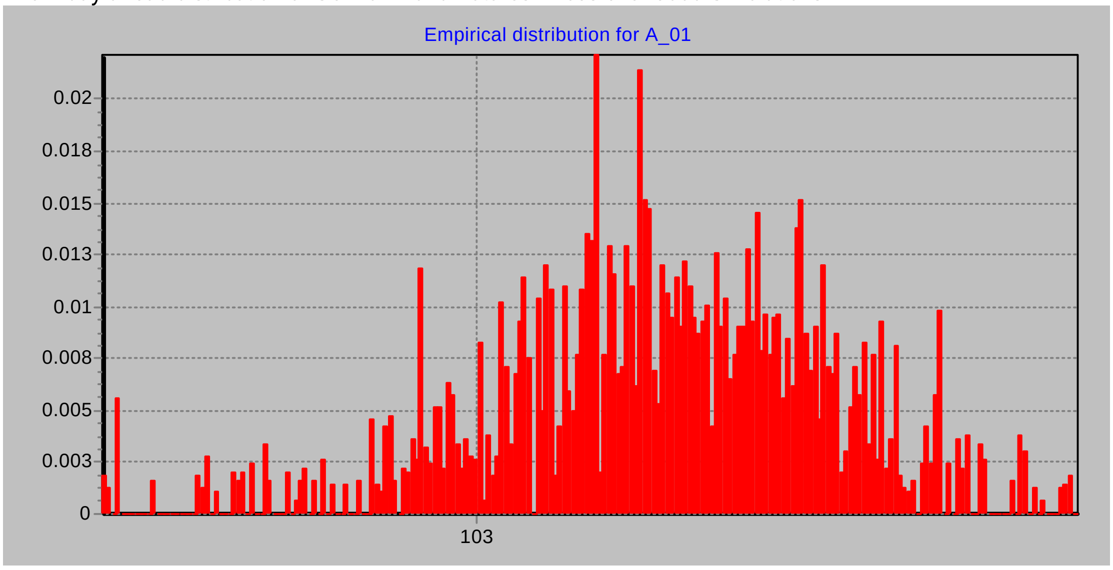
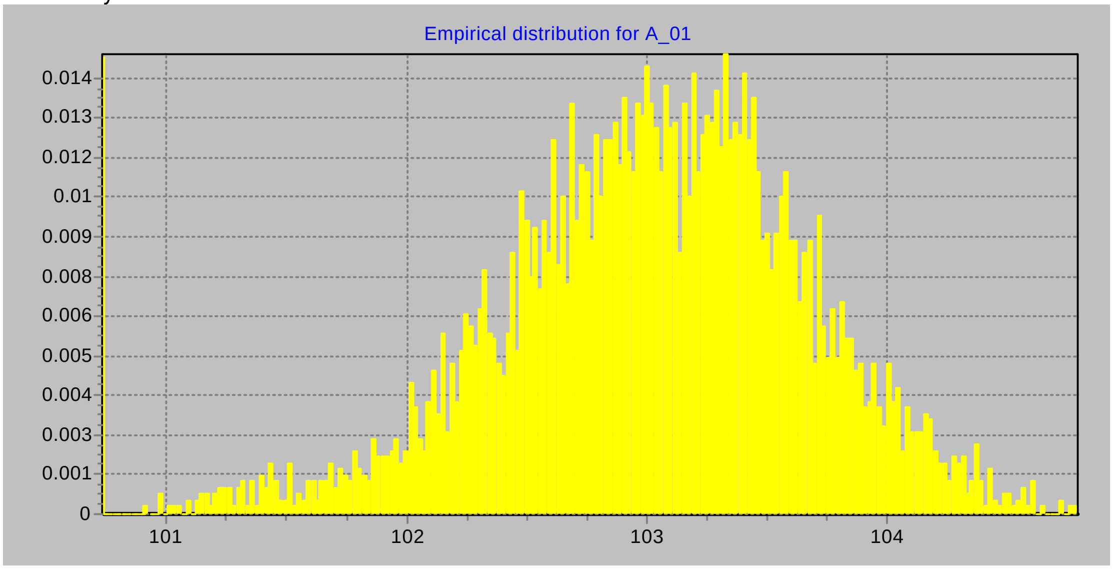


Figure 2

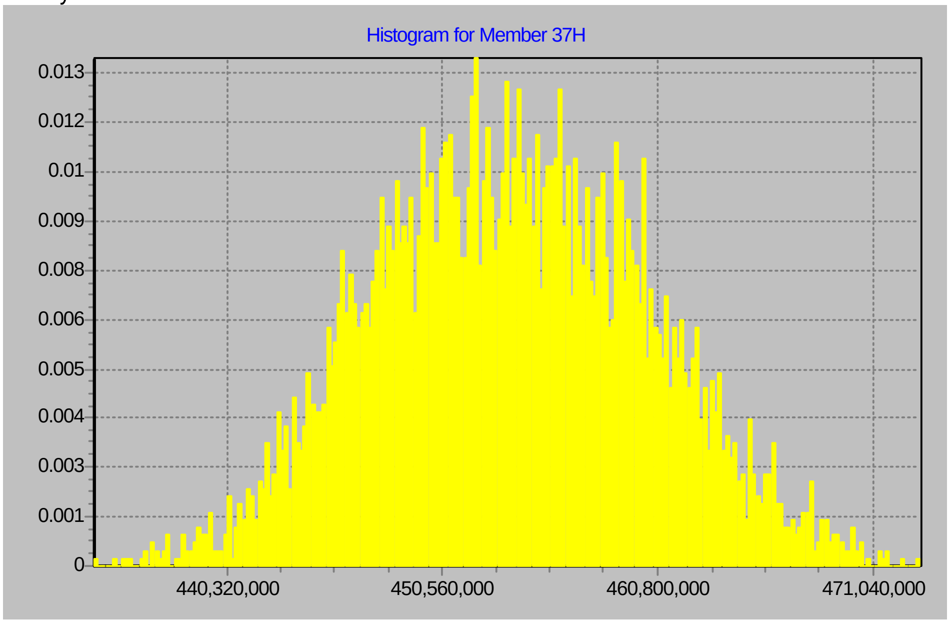
The 10-day ahead distribution of German Bund Futures Prices over 5000 Simulations



Below we obtain a distribution of portfolio values at a given time horizon e.g. 10 days. The set of gains or losses from the value of the portfolio based on closing prices (at the start of simulation) can be obtained, to give their distribution. The representative "lowest value" of the portfolio e.g. for the 99th percentile, can be compared to the value of the portfolio at the start of simulation, to obtain the 99th percentile loss. A ten-day ahead multi-contract portfolio example (a portfolio of futures and options in a variety of LIFFE contracts) is illustrated in Figure 3:

Figure 3

10-day ahead Portfolio Value Distribution over 5000 Simulations



5.1 Options

Options price paths are obtained from the corresponding asset price paths by using an options pricing model applied to each asset price in the path and other relevant option pricing parameters e.g. implied volatility, strike price, time to expiry and interest rate. For the present we keep the values of these other parameters equal to their values at the start of simulation.

Thus if the call option price is denoted c $= f(p_t, X, \sigma, \Gamma\text{-}t, r)$ (21)

where p_t is the underlying asset price at current time t, X is the strike price, σ is the implied volatility, Γ-t is the time to expiration and r the risk-free interest rate, then the price path for the call option on a given asset is

$$\mathbf{c}_{t,t+1,,t+i} \quad = \quad \mathrm{f}(\,p_t\,, \mathrm{X}, \sigma, \Gamma\text{-t}, \mathrm{r}), \mathrm{f}(\,p_{t+1}\,, \mathrm{X}, \sigma, \Gamma\text{-t+1}, \mathrm{r}),\ldots, \mathrm{f}(\,p_{t+i}\,, \mathrm{X}, \sigma, \Gamma\text{-t+i}\,, \mathrm{r}) \qquad (22)$$

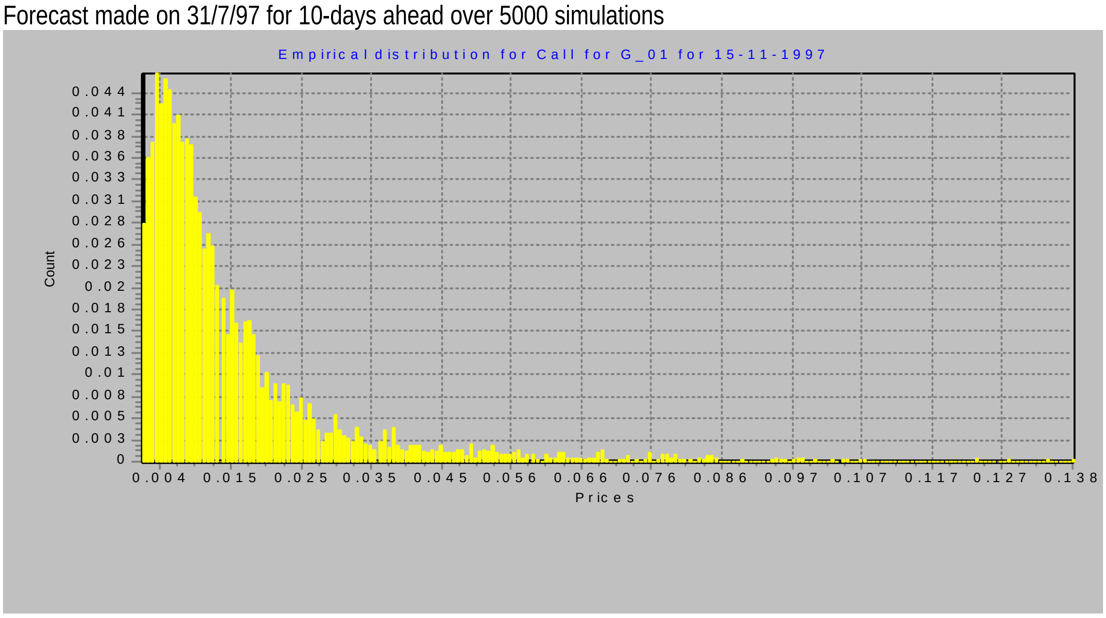
Where $p_t ,\ldots, \ p_{t+i}$ is the first vector (i.e. for the first asset) from (18).

Additional option pathways use the asset prices from the corresponding asset price vectors in (18). The option price paths are weighted as in (19) with options net lots. The portfolio pathway is given as before in (20). The put price is similarly given.

Figure 4 illustrates an example of the ten-day ahead distribution of prices for an out-of-the-money call option, for 5000 simulations, on the LIFFE Long Gilt futures contract (near month). The time to expiry was three and a half months (expiry date 15/11/97), the strike price was 123 points, the current underlying futures price was 115.44 points. The option's market price was 0.03 and the ten-day median forecast price was 0.009; the minimum price for the option was zero and the maximum of 0.138 is indicative of the non-linearity of option pricing.

Figure 4

Forecast made on 31/7/97 for 10-days ahead over 5000 simulations



6 SWAPS

An interest rate swap's value is derived from a sum of discounted cash-flows as defined by the swap's contract terms: to value a swap we require the set of zero coupon interest rates corresponding to the dates of these cash-flows.

For example for a swap with 3 future cash-flows remaining before it matures, its value is denoted by an appropriate swap valuation function of zero coupon interest rates:

$$s = g(i_1, i_2, i_3, \phi) \tag{23}$$

where ϕ represents parameters defined in the swap contract necessary to value it (e.g. coupon, floating interest rate, notional principal amount, payment dates of the cash-flows, maturity date, etc.); i_1, i_2, and i_3 are zero coupon interest rates (term structure) for dates corresponding to the future payment dates. The value of a swap at a given close of business will utilise the zero coupon the term structure at this time.

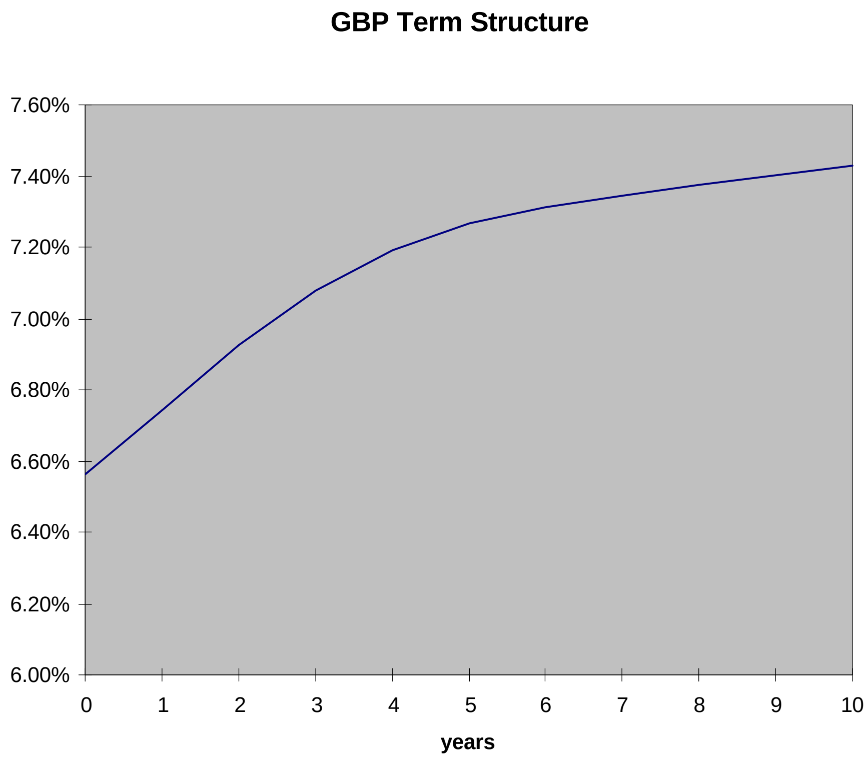
A swap value pathway is obtained from simulation of the pathways of zero coupon interest rates. The simulation of interest rate pathways for "key" rates (in this case the zero coupon rates in the description of the dataset) is described by equations (8) to (16) for the simulation of multiple pathways.

Thus for the eleven zero coupon interest rates described in the dataset, for 1 day to 10 years, each of the rates at time t in the present corresponds to asset prices p^* at time t. Equations (12), (14) and (16) describe the pathways for the first three interest rates; variances for these are described by equations (11), (13) and (15). We will require 11 sets of equations for our interest rates in each of the four currencies in the dataset. In

a given currency simulated pathways for the eleven rates constitute a simulated term structure pathway from which swaps may be valued: to obtain simulated rates which correspond to particular cash-flow dates in a swap we interpolate between the simulated key rates.[9]

Figure 5 is an example of the term structure of interest rates out to 10 years for Sterling prior to simulation, produced by linear interpolation:

Figure 5



For simplicity, if we consider that the three asset (interest rate) pathways from equations (12), (14) and (16) correspond to the cash-flow dates for our swap (no interpolation of rates required), then writing i^* for p^*, we depict the 10 x 3 matrix:

[9] The production of an interest rate term structure from simulated rates requires the use of smoothing algorithms; we examine various methods of interpolation between the "key" rates by which we define the term structure e.g. linear interpolation, cubic splines, polynomial smoothing approaches.

$$\mathbf{R} \quad = \quad \begin{bmatrix} i^*_{1.t+1} & i^*_{2,t+1} & i^*_{3,t+1} \\ i^*_{1,t+2} & i^*_{2,t+2} & i^*_{3,t+2} \\ i^*_{1,t+3} & i^*_{2,t+3} & i^*_{3,t+3} \\ \vdots & \vdots & \vdots \\ i^*_{1,t+i} & i^*_{2,t+i} & i^*_{3,t+i} \end{bmatrix} \tag{24}$$

where i = 1 to 10 days.

Each column of the matrix represents equations (12), (14) and (16) respectively i.e. they are the asset pathways to 10 days. To obtain a swap value pathway we require a row from the matrix for each day in the swap value path:

$$\mathbf{s}^*_{t+i} \quad = \quad \mathrm{g}(\, i^*_{1,t+1}, i^*_{2,t+1}, i^*_{3,t+1}, \ldots\ldots\ldots, i^*_{1,t+i}, i^*_{2,t+i}, i^*_{3,t+i}, \phi) \tag{25}$$

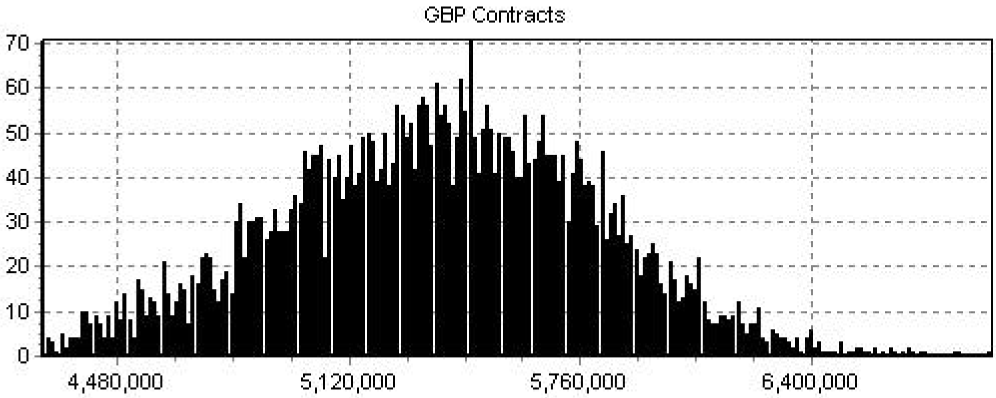
For many swaps, the swap value pathways are aggregated as described generally for any set of assets, in equations (18) to (20); the net positions w_n for swaps can be represented as +1 or -1 for each swap, to describe the payment or receipt of fixed interest cash-flows respectively. Furthermore, aggregated values for portfolios of swaps and futures and options contracts may be obtained with no fundamental change to our methodology. 5000 simulations may be performed for portfolios of swaps, futures and options, from which worst case losses can be obtained.[10]

In figure 6, we simulate 5000 values of a random portfolio of “plain vanilla” interest rate swaps in Sterling, over a 10 day holding period. The 5000 portfolio values are obtained from 5000 simulated interest rate term structures.

[10] Appropriate currency exchange rates for the given close of business are currently used in the simulations where contracts are denominated in different currencies, to convert all values to a common currency.

Figure 6:

Random Swaps Portfolio Values over 10 day holding period for 5000 simulations (y-axis is the count of values in each bar). Portfolio Values in Sterling

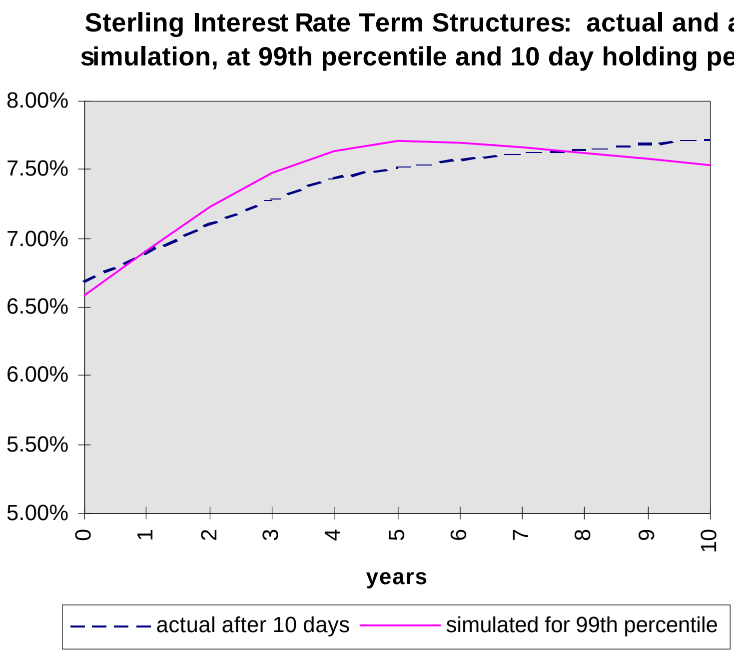


The distribution of portfolio values is shown in the histogram; the lowest value, represented by 99th percentile, is compared to the median portfolio value. This is the "worst" loss for the portfolio, equal to £1,087,421 and is the difference between the least value at the 99th percentile of £4,280,410 and median value of £5,367,831.[11]

In Figure 7 we show the simulated linearly interpolated term structure from which the 99th percentile, 10 day holding period portfolio value is calculated. This simulated term structure is compared to the actual observed term structure 10 days on from the date at which simulation was started.

[11] Alternatively the loss may be computed from the initial portfolio value rather than the median. The two losses are the same in RiskMetrics because the median is assumed to be equal to the initial value in that methodology.

Figure 7



7 CONCLUSION

Our simulation methodology of nonlinear securities allows for a fast evaluation of VaR and worst case scenarios for large portfolios. It takes into account current market conditions and does not rely on the knowledge of the correlation matrix of security returns. Our simulation is based on the empirical distribution of the data; we do not impose a particular probability function and so we do not compress the tails or alter the skewness of portfolio returns. Therefore our methodology provides reliable assessments of uncommon but possible portfolio losses.

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