Estimating the joint tail risk under the filtered historical simulation. An application to the CCP’s default and waterfall fund.

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To ensure that central counterparties ("CCPs") are safe in all market conditions the European Union (EU) has adopted legislation, commonly known as the European Market Infrastructure Regulation ("EMIR") that deal with their organisational requirements including prudential requirements in relations to margins and the waterfall and default funds. It has published in a single Regulation (EU) No 153/2013, the technical standards required to be adopted by all CCPs operating in the EU. EMIR requires a mandatory clearing of certain standardised OTC (i.e. over-the-counter) derivatives transactions through central counterparties. A risk methodology that can meet some of the most challenging technical requirements, such as sensitivity testing, estimating the probability of joint member defaults and reverse stress testing is the Filtered Historical Simulation (FHS). In this study we extend the use of Filtered Historical Simulation in estimating the potential losses the CCP would face from a multiple default. The proposed methodology provides a probabilistic estimation of defaulting of named members, the expected size of losses, i.e. the joint expected shortfall (JES), and confidence intervals around the JES. This in turn provides an estimate of CCPs need of financial resources to absorb multiple defaults. Our methodology is carrying a full re-pricing of all instruments in the portfolio and takes into account positions that expire before the profits and losses (P&L) horizon. Order statistics tell us that estimates on the tails is unreliable. To handle this risk we carry out a bootstrapping of 5,000,000 simulation trials. The bootstrapping of 5,000,000 trials is repeated 5,000 times to generated the density of the JES. .

Central counterparty risk management, filtered historical simulation, tail dependency.

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I. Introduction

Central Counterparties (CCPs) stand between two clearing members in a bilateral transaction and mitigate the credit risk in case either fails to meet its marking to market obligations. In recent years the CCPs expanded their activities in over-the-counter derivatives and other financial products. In 2011 the CCPs cleared 50% of the $400 trillion of outstanding (notional) interest rate swaps (IRS) and under 10% of the $30 trillion of the credit default swaps (CDS)\(^3\). The Regulation (EU) 153/2013 provides a technical description of the risk management procedures that any CCP operating in the European Union needs to follow\(^4\). The scope of article 53 is to stress test the CCP’s internal and external financial resources in meeting a multiple default of the largest accounts. The scope is extensive as a CCP must define its internal risk policy.

Nowadays the CCPs have different controls to manage their counterparty risk. At the first level, the participating clearing members are carefully selected according to high quality credit rating criteria. The second tier of risk management instruments available to the CCPs are the initial margin and variation margin. These are referring to a particular type of contract and their size is often measured as a fraction of the notional value of the contract. Both are largely affected by the volatility that characterizes that underlying asset as well the prevailing market conditions\(^5\). The aggregated daily margin for a member is met by cash or qualified collaterals. The selection of the various types of collaterals is another risk management tool available to the CCPs.

It can be assumed that the probability to default, due to activities exogenous to the trades cleared with the CCP, it is equal for each member because at tier one only members with high credit ratings are accepted to participate. The second tier of risk management tools, the margin held in each account, is a tool to handle the exposure to market risk of the member portfolios. As adverse market conditions will cause member losses to reach the variation margin the CCP will issue a margin call, an

\(^3\) Source, Heller and Vause (2011) p 68.

\(^4\) These minimum technical standards represent a benchmark that any CCPs outside the EU should meet in order to be approved by the Member States national authorities. A pre-condition for any bank domiciled in that Member State to be permitted to trade in that non EU CCP.

\(^5\) In 2009 the G20 leaders decided the OTC derivatives would be cleared through CCPs. This led to an increase of the volume cleared by the CCPs and therefore to a raise of the daily margins met by the clearing members. The members were asked to provide more high quality collateral to meet these augmented margin requirements.
ultimatum, asking the member to deposit additional cash or collateral to cover these losses. In practice, margin calls could happen every day but these are random, usually of small size, and are met by the members. In the event a member fails to meet the margin call the CCP inherits its positions and a mechanism of liquidating this account, by hedging, transferring or closing out positions, is activated. In this study the term default will refer only to the event where a member fails to meet his margin obligations, with a single CCP, and where this is caused by the market risk that his trades are exposed to.

In case margin does not cover the losses in the defaulting members account between the time of the defaults, at which point the CCP inherits its positions, and until their closeout, a third tier of risk management tools, the CCPs default fund and ultimately the CCPs own equity, are deployed in a defined order to facilitate the winding up of the defaulters’s positions. The CCP is facing the catastrophic risk that one or more member defaults and the aggregated liquidation losses are in excess of these members initial margins, the default fund and its own equity. In that case the CCP becomes insolvent, but there may be in place an insolvency mechanism to absorb, even part of, the exceeding losses, see Lazarow (2011).

Lazarow (2011) in a survey of the current trend on risk management mechanisms followed by the largest CCP’s around globe, reports that the target is to cover large defaults with defaulter’s margins and contributions to the default fund and potentially the CCP own equity. Only when large multiple defaults occur additional financial resources may be necessary to be deployed. In the first place the non-defaulting member’s contributions to the default fund will be deployed. As expected, in case the default fund cannot cover all of the losses the CCP itself will default. Therefore, the exceeding losses should be financed by the non-defaulting members and the other creditors of the CCP, see Elliott (2013). The Regulation is not specifying how the default fund should be financed but the most common source of financing is by mutualisation.

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6 The first step in administering the default is to stabilize the portfolio so that it is hedged as far as it is practicable. The positions may then be considered for auctioning off, using the available funds as required. Buying and selling positions in the market to neutralize them can also take place. The defaulting member’s personnel may be used as appropriate. As the portfolio is “dismantled”, re-hedging can be necessary, as the simple concept of closing positions more or less simultaneously would probably not obtain, and risks would change as parts of the portfolio are disposed of. Initial margin then provides additional cover. Only margin from the defaulting member may be used for a default i.e. initial margin is a “returnable deposit” and one members margin cannot be used to cover another member’s default.

7 Pirrong (2011) quotes as available tools to fund losses due to a member default the netting, collateralization, insurance, equity
As for the Regulation, the framework and governance stipulations require that a minimum size of the default fund be established but the CCP can implement its own policy framework, by defining extreme but plausible market conditions, to stress test against the strength of the default fund. At the core of the stress testing in the Regulation is the historical simulation, to be performed on the most recent 30 years of data.

The historical simulation (HS) is a popular method for scenario generation among the banks and was applied independently by Hendricks (1996) and Barone-Adesi et al. (1996). Hendricks\(^8\) uses unconditional series assuming a constant volatility world during the observation period. He recognizes that in such a small sample extreme quantiles such as the 95% and 99% are difficult to estimate accurately and he rightly observes that risk measures for longer periods do not obey the square-root-of-time rule. In Barone-Adesi et al (1996) the aim is to measure a portfolio’s risk exposure by bypassing the correlation matrix, widely used by the banks at that time, without ignoring some common characteristics of the financial data\(^9\). They mapped current portfolio positions to form a synthetic series of past returns that could reflect its behavior over time arguing that “...past returns contain the necessary information about the current portfolio’s risk return trade-off”. They captured the volatility clustering present in the portfolio, that was caused by the heteroskedasticity of the assets components, by fitting a conditional GARCH on the historical returns of the synthetic portfolio, “…thus, we can obtain volatility estimates for the portfolio by studying directly its own past returns rather the returns of its components”.

Barone-Adesi et al (1998) proposed the univariate filtered historical simulation where the historical portfolio’s returns, standardized by its conditional volatility, were used as innovations in the GARCH process to form conditional recursive pathways for the conditional mean and variance. Barone-Adesi et al (1999) extended the filtered historical simulation to a multivariate, multi-period setting that could handle nonlinear positions, stemming from derivative contracts. This adaptation allows for the handling of expiring positions that cause a significant change of the

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\(^8\) Hendricks (1996) discusses this approach followed by some banks to stress test their risk models and calls it as historical simulation. See next section for further discussion on the historical simulation.

\(^9\) For a discussion among others see Allen and Satchell (2014). Cont (2001) classifies the characteristics of speculative price changes into eleven types.
portfolio density function, which in turn could have had a large impact on the portfolio risk in the post-expiration horizon. A backtesting analysis on real clearing member portfolios, revealed that for some member accounts that were heavily weighted in expiring out-of-the money options, the ten-day risk exposure by far exceeded the one day percentile of simulated profit and losses (P&L) adjusted by the square route of time, see Barone-Adesi et al (2002). A simple case in which this occurs is when the portfolio is an at-the-money straddle. To our knowledge, even today CCPs risk models are based on scenario analysis on a one day holding period and risk estimates on longer holding periods are obtained by multiplying the former by the squared root of the time.

The Regulation considers only the impact that the liquidation of the large member accounts have on market prices, see article 53 points 3 and 4. Moreover, the impact a default would have on the market contagion should be considered, that is a second member may default days after the first defaulted member. Still, the Regulation is concerned that historical scenarios may not fully reflect potential market movements under current market conditions and requires the generation of hypothetical but plausible, scenarios that go beyond the historical extremes to reflect a wide range of potential stress conditions (article 32) for the portfolios under examination.

The oldest scenario model currently in use is SPAN, the Standard Portfolio Analysis of Risk, developed originally by the Chicago Mercantile Exchange (CME) in the 1980’s. As a methodology SPAN has serious limitations for present-day use as envisaged under the Regulation. Although sets out with parameter values set as a number of standard deviations, it has a limited number of scenarios, a complex system of spread risk and inter-contract credits, and sums margins across contracts. There can be no probabilistic inference related to the final margin for complex portfolios. The conceptual simplicity of SPAN limits its use to initial margin calculations only, for certain markets, which has been its use to date and for which use it was originally designed. SPAN offers no assistance as a methodology for estimations beyond this i.e. for the default fund and for the extensive requirements set by the Regulation. For a detailed discussion on SPAN see Cotter and Dowd (2006). Despite its shortcomings SPAN was for long time the only risk model operated by the CCPs in clearing traded options and futures.

In this study we develop further the filtered historical simulation to measure the tail dependency in the P&L between member accounts. Extreme market movements are not set in an ad hoc way but rather are formed by increasing the number of simulation trials. We conclude this study with a simulation on three futures series as used in Giannopoulos (2008), that exhibit tail dependency. By increasing the number of simulation trials to 5,000,000 we accurately emulate the density in the tails to
obtain efficient multivariate trustworthy risk estimators such as the probability of a joint default and the implied correlation on the tails even at the very high confidence level of 99.9%. Having an insight on the size the aggregated member losses could reach, the CCP is in a better position to strike the optimal balance between marking to market each account and the size for the default fund. For example the size of the default fund could be set as the difference between the simulated aggregate maximum losses the large members are expected to face and the initial margins of the same members held by the CCP. Furthermore, we estimate the overall financial resources the CCP will need in the second and third risk management tiers to handle successfully joint defaults of the largest portfolios, while allowing the CCP, within its internal risk policy, to strike the balance between initial margin and default fund. We show a flexible adaptation of the filtered historical simulation that can incorporate the market risk of any collateral securities the member may have put forward to meet its margin obligations

II. Traditional tail dependency models.

The Regulation follows a top-down approach and only outlines the criteria for assessing the performance of the risk models used by the CCPs. Does not specifies any characteristics of these models should meet neither is questioning what models the CCPs have implemented in capturing the dependency among the risk factors. Nevertheless, the Regulation is concerned about the increase in tail dependency during high volatile periods, especially during market downturns. The presence of non-normal dependency in asset returns, i.e. the correlations are stronger during market downturns than upturns, is reported among others in Erb et al (1994), Longin and Solnik (1995, 2001), Ang and Bekaert (2002). Indeed, the Regulation in chapter VII addresses the possible negative impact of any sudden up-rises in correlations among risk factors on the value of member accounts, and requires the CCPs to test the reliability of correlations over a lookback period\textsuperscript{10}.

Risk estimates of multiple defaults, such as the joint default probability, the default correlation and the size of the aggregated losses of the defaulting member accounts require knowledge of the dependency on the left tail of these accounts. Traditional measures of dependency such as the covariance are not appropriate because are based on the averaging or centering of the distribution and not measuring the dependency on the extremes. An appropriate measure could be the conditional

\textsuperscript{10} Shifts in correlations can also be due to “model risk”. Kerkhof et al (2010) attribute the “Model risk” to any of the following components: estimation risk, misspecification risk and identification risk.
probability of observing a large extremal drop in the value of a member’s portfolio, given a similar extremal change in the value of another’s member portfolio,

\[ e_{ij}(q) = P(P_i < q_i | P_j < q_j) \]

where \( q_i, q_j \) are the extreme quantiles, e.g. 1% or 0.1%, on the left of distribution of the portfolios for members i and j.

In extremal analysis typical models are the copulas and the multivariate extreme value theory (EVT). Early work on the multivariate extreme value theory (EVT) in risk management is that of McNeil (1999) and Hauksson et al (2001). The univariate version of EVT has found some applications in financial risk management, such as in operational risk. There is some unconfirmed reports that some CCP’s are firstly generating a small number of one day ahead scenarios with a simplified version of filtered historical simulation, which we will refer to it as quasi filtered historical simulation, and thereafter are using a EVT to model the density of extreme quantiles of the portfolio accounts. Computational issues are making not feasible the use of EVT on multivariate extremes of large dimensions. To our knowledge there is no evidence of any use of the multivariate EVT in estimating the magnitude or the probability of a multiple default.

Copulas are a flexible method for modeling the dependence structure of financial time series as they relax the assumption of normality and combine the marginal distributions with the copula function to produce a multivariate joint distribution and capture the dependency among the variables. Copula models represent a promising alternative to the multivariate normal model without facing the criticisms of latter for its assumptions of multivariate normality of joint distribution of the various risk factors and the use of the correlation matrix to measure such dependence. Early work on multivariate copulas in finance include Embrechts et al (2002) and Cherubini et al (2004). The use of copula in estimating the VaR of a portfolio of only two assets is discussed in Palaro et al (2006). Kole et al (2007) show how to select the right copula in a multivariate framework with the use of fitting tests. Still, these empirical investigations are limited to a linear portfolio of three assets; at higher dimensions there are unmanageable computational constraints, a fact does not render copula methods suitable risk measurement tools to the CCPs. However, the estimation of the tail dependency through the copula modeling relies on the correct specification of the full density of the variable. Nonetheless close form solutions for many of the joint densities cannot be derived. Estimations based on numerical methods such as maximum likelihood do not provide any insight on the distributional properties of the extremal estimators.

Brechmann et al (2013) use a pair copula construction with a D-vine copulas in an application of portfolio risk management of a 52-dimensional data set of the Euro
Stoxx 50 index. Their empirical findings indicate that vine copula are performing better in forecasting the VaR at the 99% but not at 95% or 90% confidence level. Despite the relative increase in the portfolio dimension to fifty assets, it is relatively small in size when compared with some of CCP’s members account\textsuperscript{11}. In addition, there is no evidence how the copula family of models could measure the market risk of some member portfolios that are heavily invested in derivatives.

The copula modeling has found use in another related field, in the credit risk and in particular in estimating default correlation in the credit default swaps, see Li (1999). However, bad applications of the Li’s copula formula are blamed for the CDO crisis in the late 2000’s, see Salmon (2009) and MacKenzie et all (2012). In particular it was used to give AAA ratings to senior tranches, that under the normal assumption should had essentially no chance of having losses. The first application of copula in risk analysis of the CCPs is found in Cumming et al (2013). They apply multivariate copula in studying the adequacy of financial resources of CCPs during multiple defaults of the largest accounts. Their analysis is all hypothetical, not calibrated to real portfolios. It also assumes a heavy tail stationary distribution, while portfolios change over time. Finally, from structured products you get the priced correlation risk. It may differ from the physical correlation.

Quintos (2001) wrote the moments of co-exceedances in terms of tail indices to build a measure of extremal correlation that doesn’t require the knowledge of the joint copula function. Bae et al (2003) used a multinominal logistic regression to measure financial contagion across markets as co-exceedances. These findings, however, are of limited use to the CCPs since the small data set, 2283 daily observations only, strained the upper/lower thresholds to the 5\textsuperscript{th} /95\textsuperscript{th} percentiles. A Monte Carlo simulation used to estimate confidence intervals relied on strong assumptions of the returns generating process such as Gaussian multivariate or students-t distribution.

\textbf{III. Generating scenarios under the filtered historical simulation}

In determining the amount of financial resources needed to cover a multiple default, and consequently the size of the default fun, it is of paramount importance the scenario mechanism upon which the CCP relies to generate the “extreme but plausible” market conditions. When in late 1990’s the CCPs began clearing over the

\textsuperscript{11} The ICE is handling about 1200 different futures and when there are added the different maturities and different maturities of interest rates, modeled in the clearing of interest rate derivatives, could reach several 1,000’s, Source ICE web site.
counter products the estimation of the swap book quantiles required the generation of scenarios across the yield curves of the major currencies, see Barone-Adesi et al (2002). In setting the variation margin of the swap book of the member, the CCP had to model the switches of the yield curves in each currency. While theoretical movements in prices may in isolation be considered stresses for some types of portfolios, this is not necessary always the case. For example simulated parallel moves and bending in the interest rate yield curve are not sufficient for assessing risk in portfolios of interest rate swaps, see Barone-Adesi et al (2002). Furthermore, the extreme but plausible scenarios must reflect the liquidity of the instruments in portfolios and so the P&L (profits and losses account) and must last more than one day. There is to be a range of future scenarios founded on consistent assumptions regarding market volatility and price correlations across markets, and the extent to which extreme price moves could simultaneously occur across markets, see for example the article 30 of the Regulation. The Regulation requires that the largest member account be stress tested against thirty years of historical prices. Implicitly it assumes that i) default occurs only on the next trading day, ii) the last thirty years of price history contain all the plausible scenarios, iii) the sequential occurrence of large price movements is not considered to be a plausible multi-period scenario.

The filtered historical simulation combines past scenarios to simulate the distribution of the member’s portfolio in the N-days holding period. The multi-step simulation allows the generation of a large number of scenarios from a smaller set of historical prices, necessary to obtain efficient extremal statistics on the forecasted density of the member portfolio. Unlike the stress testing outlined in the Regulation where a history of thirty years could generate about 7,500 of unique scenarios, in filtered historical simulation the number of unique scenarios is $N^D$ where $N$ is the number of observations in the historical dataset and $D$ the number of days in the holding period. These unique scenarios represent any possible sequential combination of the returns in the historical datasets. This is because of the multiplicative impact on the extreme scenarios present on the historical dataset the filtered historical simulation reaches wider extremes to those present in the historical data. Huang Yuan (2012) correctly observes that filtered historical simulation “…provides a systematic approach to generate those extreme scenarios that are not included in the scenario of history,..., needs less historical records than the historical simulation method...to simulate the two tails”. Just a few years of daily, historical data are sufficient to assess the risk of extreme movements of member portfolios.
IV. Estimating the size of the CCP financial resources

The first tier of risk management measures, the initial margin, is also the front line of defense to handle the exposure the CCP is facing in the event of a member’s default. Setting the margin too high will reduce the probability there is a margin call which could result in the member failing to meet his obligations and so defaulting. Contrariwise, high margins implies high opportunity costs for the members, see Booth et al (1997). In the relevant literature they are three main approaches proposed for setting the initial margin. i) a statistical approach to calculate the each contract margin that is often adjusted by the subjective judgment of an expert to reflect information not present in the historical dataset, see Figlewski (1984) and Gay et al (1986), ii) optimization models of an efficient prudential margin management policy which minimizes the account overall margin, settlement, default and any other costs, see Dewacher et all (1999). A third approach, proposed by Barone-Adesi et all. (2002) calculates the overall portfolio “benchmark” margin as a percentile of the simulated portfolio values.

With the recent regulation focusing on the adequate financial resources the CCPs must have to be able to handle the insolvency of one or more larger members, there has been a new direction in the literature towards modeling the initial margin jointly with the default fund. Nahai-Williamson et all (2013) argue that because the Basle rules12, aiming to control the banks’ risk exposure to the CCP’s, apply different risk weights on initial margin and the default fund, that is 2% against nil, this may have an impact on the CCP’s policy in setting the balance between initial margin and default fund. These authors model the optimal mix between initial margin and default fund as objective cost minimization for non-defaulting members. They estimate the mix between initial margin and default fund as weights but they do not estimate the total size of the financial resources need by the CCP. A regulator should target more cost minimization to the markets, that is the loss given CCP failure. In addition, they never discuss correlation. Higher rated members are likely to have higher default correlation, reducing the mutualisation advantage.

Cumming et al (2013) proposed a semi-parametric methodology in quantifying the level of initial margin and default fund. They construct hypothetical portfolios representing members with leveraged positions and use EVT techniques to model the distribution at the tail of these portfolio simulated returns. They fit a copula on the tail of the empirical distribution of member portfolios. The joint default risk is estimated as a bivariate copula with the premia paid on the tranches of different

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12 See “Capital requirements for bank exposures to central counterparties, Basle Committee on Banking Supervision, Bank of International Settlements, July 2012 and April 2014.”
structured products to provide the default correlations in the bivariate copula. The authors acknowledge that their methodology relies on unrealistic assumption related to the creation of the scenarios and equal probability of a member to default under each scenario.

This study investigates the filtered historical simulation as a framework for evaluating the adequacy of CCP’s financial resources in handling multiple defaults. A typical CCP will use the financial resources to cover any resulting losses in the following order\(^{13}\): i) the defaulting member initial margin, ii) the default fund contributions of the defaulting member’s, iii) the CCP’s surplus fund, a portion of its own share capital, iv) the default fund contributions of solvent clearing members, v) additional contributions of solvent members and possibly an insurance cover. In this study we divide these resources in two main categories, the initial margin (IM) and default fund (DF). The former is made up by any kind of financial resources that are provided by the member itself or the CCPs owned funds, classes (i) to (iii). The latter is formed by the resources in classes (iv) and (v), the mutualized financial resources.

Under the filtered historical simulation the risk the member imposes to the financial resources of the CCP could be estimated from the forecasted density distribution of its portfolio, \(P_{i,t+j}^* \in \Theta_{i,t+j}\), where \(i = 1,2,3 \ldots, M\) is the number of simulated scenarios and \(j\) is the number of days in the holding period. A profit or loss scenario (P&L), \(L\), for the member portfolio over the \(j\) days of the holding period is given by \(L = P_{t+j}^* - P_t\). The CCP has the discretion to set the rules that determine the level of financial resources committed by each member. The level of the first set of resources contributed by each member, the ones that are aiming to cover any losses arising by its own default, the IM, could be set equal to the tail threshold \(\ell\), or quantile, on the member’s portfolio forecasted density for the period \(t+j|t\),

\[
IM = \inf\{\ell: P(L \leq P_t) \leq (1 - \alpha)\}
\]  

(1)

The threshold, \(\ell\), can be set to the level over which the member portfolio losses are expected to be contained. Thus, the probability that at the end of the holding period the member will face a loss \(L\) larger than the quantile \(\ell\) is less or equal to a predefined confidence level \(\alpha \in (0,1)\).

This approach follows the margin calculation method proposed by Barone-Adesi et al (1999) which allows for a dynamic assessment of the market risk of a member’s portfolio by capturing jointly the price and volatility dynamics of the underlying as well any co-dependences with other underlyings. The implied volatility of any

\(^{13}\) For a detailed discussion on the financial resources the major CCP’s have at their disposal and the order they will use the various types in a case of a member default, see Chamorro-Courtland (2011).
options in the portfolio could be modeled as conditional on the volatilities of their underlying.

When \( \alpha > 0 \) on average it should be a margin call every \( \frac{\text{No clearing members}}{\alpha} \) days and normally it will be met by the member. In the event the defaulting member losses exceed \( \ell \), the CCP technically defaults, and the liquidation losses in excess of the member’s IM are covered by the remaining set of available financial resources, the contributions of the non-defaulting members and any other sources, DF.

Under the filtered historical simulation parallel scenarios are created for all clearing member portfolios. The generation of simulated values for the \((P_1, P_2, \ldots, P_I)\) vectors of portfolios is obtained by aggregating the pathways of the values of the contracts in the portfolios accounts. The first step consists of generating a set of simulated pathways for each contract. The intuition behind the filtered historical simulation is that the innovations fed in the price/variance equation that best describes the behaviour of each contract are drawn from a set of residual returns of the contract itself. These are then filtered to become identically and independently distributed, removing serial correlation and volatility clusters. For example, assuming a GARCH(1,1) process with both moving average (\( \theta \)) and autoregressive (\( \mu \)) terms, our estimates of the residuals \( \varepsilon_t \) and the variance \( h_t \) are:

\[
\begin{align*}
    r_t &= \mu r_{t-1} + \theta \varepsilon_{t-1} + \varepsilon_t \\
    \varepsilon_t &\sim N(0, h_t) \\
    h_t &= \omega + \alpha (\varepsilon_{t-1} - \gamma)^2 + \beta h_{t-1}
\end{align*}
\]

(2.a) (2.b)

The residual returns \( \varepsilon \) purified, of any ARMA effects, are random but their distribution is not stable. The standardised residuals \( e_t = \varepsilon_t / \sqrt{h_t} \) are stationary and random, and could be used as innovations in a simulation. The simulated price for a single risk factor and N day holding period is given by

\[
\begin{align*}
    P_{t+1}^* &= P_t (1 + \hat{\mu} r_t + \hat{\theta} z_t + z_{t+1}^* ) \\
    P_{t+j}^* &= P_{t+j-1} (1 + \hat{\mu} r_{t+j-1}^* + \hat{\theta} z_{t+j-1}^* + z_{t+j}^* ) \quad \text{for } j > 1, \ldots, N
\end{align*}
\]

(3)

Where \( z_{t+1}^* \), given by \( z_{t+1}^* = e^* \sqrt{h_{t+1}} \), is the innovation term in the mean and variance equation is a randomly selected standardised residual \( e^* \) rescaled by that scenario’s volatility forecast. The scaling ensures that the random historical –filtered-return reflects the current market conditions. Scenarios for the conditional mean the conditional variance in (2) are simulated in parallel until the last day of the holding
period. The equation in (2) is updated at each day to produce pathways for the prices of the contract, $fx$, or interest rate factor.

The correlation between different assets is modelled implicitly by the random drawing of strips of residuals i.e. for a given pathway and day in the holding period (“node x,y”), each separate asset’s simulated price and volatility is produced from standardised residuals at the same date. Hence the empirical price co-movements between assets are represented across corresponding nodes in volatility and price pathways. The price pathways for assets in a portfolio can be aggregated, to produce portfolio price distributions, without resorting to the correlation matrix, or assuming any particular distribution for the data.

In a filtered historical simulation the member portfolio density forecast is formed by millions of scenarios providing a solid structure of the tail, unique for each member portfolio. The idiosyncratic dynamics between a set of portfolios has also an idiosyncratic structure, which is characterised by the implied co-dependency of historical returns of the underlying assets as well the market conditions at the end of the last trading day. The level of each exceedance will depend on the volatility of the underlying assets that a member has traded upon as well on the type of collaterals that the member has deposited with the CCP. The filtered historical simulation is flexible enough to combine parallel scenarios for the collaterals, producing a more accurate estimation of the level of losses the CCP could face after the liquidation of the member portfolio. The filtered historical simulation provides an ideal platform to create “extreme but plausible” scenarios, as required by the Regulation, to stress test the size of the default fund of the CCP.

A joint default occurs when in a simulated-scenario $i$, two or more members face losses in excess of the threshold $\ell_i$, as

$$
\begin{align*}
L_{i=1,t+j}^* & < \ell_i P_i^* \\
& \ldots \ldots \ldots \\
L_{i=n,t+j}^* & < \ell_i P_i^*
\end{align*}
$$

(4)

These scenarios derive from historical returns and so are plausible. Yet they are extreme because they have a very small probability of occurring. By identifying the number and magnitude of those scenarios where the losses of the largest portfolios could exceed the margin held by the CCP, a number of tail risk gauges can be derived. In approaching the expression in (4) from a statistical perspective it is required to develop a model that captures the extreme left tail dependency of the P&L of the member portfolios without being prone to the drawbacks mentioned.

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14 For a detailed description of the algorithm see Barone-Adesi et. al. (1999).
earlier. An efficient algorithm that could be employed to build such a model could be the empirical copula of Deheuvels (1979).

Let \((P^*_i, ..., P^*_n)\) being the set of \(n\) simulated –random- values of a given member portfolio on \(j\) days holding period beginning on closed of business at date \(t\). The empirical marginal cdf for the \(i\)th portfolio, \(P^*_i\), is given by

\[
\hat{F}_i(P^*_i) = \frac{1}{n+1} \sum_{v=1}^{n} I(P^*_{iv} < \ell P_i)
\]  

Where \(I(\cdot)\) denotes the indicator function that at each filtered historical simulation scenario could take either a value of 1, if there member account losses exceed the threshold (margin), \(P^*_{iv} < \ell P_i\), and 0 otherwise. The denominator \(n + 1\) ensures that the cdf for the random variable \(F\) is always less than one. The empirical copula is given by

\[
C_{emp}(u) = \frac{1}{n+1} \sum_{v=1}^{n} I\left(\left(\hat{F}_1(P^*_v) \leq u_1\right), ..., \left(\hat{F}_d(P^*_v) \leq u_d\right)\right)
\]  

Where \(u_{iv} = \hat{F}_i(P^*_{iv})\) for \(i = 1, \ldots, d; v = 1, \ldots n\). is the empirical probability integral transforms for the vector \(u = (u_1, \ldots, u_d)\), using the marginal cdf’s. By rearranging (10) the probability for a multiple member default, JES, is given by

\[
C_{emp}(JES) = \frac{1}{n+1} \sum_{j=1}^{n} I\left(P^*_1 < u_1, ..., P^*_d < u_d\right)
\]  

The model in (10) captures the multivariate distribution of tail exceedances of the member portfolios independently of their margin. Unlike the parametric copula where the knowledge and adjustment of the right copula function that best fits the data is required, the empirical copula is an efficient estimator of the true copula, see among others Tsukahara (2005).

Knowing the probability of an event where two members defaulting on the same scenario, the implied correlation \(r_{A,B}\) can be derived as the expected value of the indicator function associated with the occurrence of that event. Moreover the distribution of the value of the indicator function is binomial. Therefore,

\[
E(AB) = E(A)E(B)+\text{COV}_{A,B}
\]  

Given \(p_{A,B} = p(A|B)\) we can express the tail correlation as

\[
p_{AB} - p_A p_B = \text{COV}_{AB} = p_{A,B} \sigma_A \sigma_B
\]  

The variance of the events that losses exceed the threshold \(\alpha\) is given as

\[
\sigma_A = \sigma_B = \sqrt{\alpha(1-\alpha)}
\]
Where $\alpha$ represents the predefined account margin which could be set equal to a small fractile on the left tail of this account simulated P&L by (2). The probability for member’s B account is facing losses in excess of its margin is given by

$$p_B = I(1)_B = \Rightarrow E_B(1)$$

(11)

Where $I(1)$ denotes the indicator function of account A. Likewise, the probability that member’s B account is facing losses in excess of its margin is given by

$$p_B = I(1)_B = \Rightarrow E_B(1)$$

(12)

$$E_{AB} = E_A E_B + COV(I_A I_B)$$

(13)

It follows that the tail default correlation between members A and B is given by

$$r_{A,B} = \frac{COV_{AB}}{(1-\alpha)(1-(1-\alpha))}$$

(14)

V. Empirical analysis

The ability of the filtered historical simulation to predict the member portfolio exceedances has been empirically examined on real member portfolios in Barone-Adesi et al (2002). Due to the lack of proprietary data in this study we will use a set of three underlying assets, which tend to have a high tail dependency. The data series used are rolled returns of the front month futures for the CL (New York crude oil), HO (heating oil) and HU (unleaded gasoline). The futures contracts are trading in NYMEX and represent the underlying assets for spread options indicating a high degree of dependency, among other see Giannopoulos (2008). After fitting an appropriate ARMA-GARCH in each series we fed the returns in (2) to generate 5,000,000 simulated scenarios for a holding period of five days.

In table 1 are shown the three cross pair probability of joint default and implied tail correlation at various confidence levels, ranging from 99.00% to 99.90%. The last column shows the probability of joint default for all three assets. In the table 2 are reported the probability of a joint default for the two largest member portfolios at $t$. In the last column is shown the associated joint expected shortfall, as a percentage of $\ell_A + \ell_B$. In the same table are shown the results for various values of $\alpha$. Estimates on the extremes have large standard errors, which renders it necessary to have a large number of trials in any simulation exercise, see David and Nagaraja (2003).
We run 5,000 repetitions of 5,000,000 trial each of filtered historical repetitions to get the empirical density of the JES for the various combinations of the three series. At 99.9% confidence level there is a chance of 0.025% of having a triple default with an estimated exceedance of 2.38% of the aggregate of the three thresholds. The standard deviation of the density of the JES estimates is shown on table 2. The third and fourth moment distribution, not shown, was very close to the 0 and 3 indicating that the distribution is normal. However, in case the portfolio is over weighted with out of the money options or contracts that expire before the end of the five day holding period, the distribution of the JES will not be known.

Extreme market conditions may affect adversely the market value of the collaterals the defaulting member has deposited with the CCP to meet its IM. Highly correlated collaterals with the member portfolios should increases the joint ES and thus the level of mutualized financial resources DF that will needed to be available to meet possible default of these members.
Table 1A. Joint default for CL, HO & HU futures. 5,000,000 simulation trials **Lower tail**.

<table>
<thead>
<tr>
<th>Conf. Lev</th>
<th>CL-HO</th>
<th>CL-HU</th>
<th>HO-HU</th>
<th>CL-HO-HU</th>
</tr>
</thead>
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<td>0.402</td>
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</tr>
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Table 1B. Joint default for CL, HO & HU futures. 5,000,000 simulation trials **Upper tail**.

<table>
<thead>
<tr>
<th>Conf. Lev</th>
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<th>CL-HU</th>
<th>HO-HU</th>
<th>CL-HO-HU</th>
</tr>
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Table 2A. Statistical inference of the JES by 5,000 simulation repetitions of 5,000,000 trials in each simulation. Lower tail.

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<th>JES %</th>
<th>Std%</th>
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Table 2B. Statistical inference of the JES by 5,000 simulation repetitions of 5,000,000 trials in each simulation. Upper tail.

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VI. Conclusions

The Regulation 153/2013 set the minimal technical standards the CCPs have to meet before being authorized to operate in the European Union arena. The Regulation addresses the need for a prudent calculation of the CCP’s internal and external financial resources needed to meet a multiple default of the largest member accounts. In this study we investigated the use of the filtered historical simulation in calculating the tail risk and tail dependency among two member accounts. The results are promising as the model could be generalized to handle a larger set of accounts. In addition the model could be modified to handle the risk of contagion due any one member default.
References


Huang Yan (2011), Calculation of Expected Shortfall via Filtered Historical Simulation, Upsala University, MSc thesis.


Cultures of Modelling


Appendix I

The Regulation goes further in setting minimum rules for reviewing and testing the various risks models adopted by the CCP. It makes frequent references to technical terms such as “statistical”, “dependency”, “correlation”. Undoubtedly the Regulation has a strong preference towards the statistical modeling and analysis. Carefully, it is not naming any specific risk measurement model but it emphasizes the need to consider “scenarios” that go beyond normal market conditions, e.g. see article 57. It abstains from making any reference to the normal or other theoretical distribution that could describe the density of the risk factors or the generated values of the member portfolio. The frequent reference to extreme scenarios and the absence of terms such as “standard deviation” is, in our view, a strong indication that the Regulation a) recognizes implicitly the fact that the density of the speculative price changes have large tails, and b) the normal distribution is inappropriate for measuring the large price movements of the risk factors. The Regulation makes frequent reference to terms such as “historical data” and “historical period”, a sign that prefers non-parametric approaches in capturing the stylized facts of the risk factors. Non-parametric models employ past prices to build the system of equations in a simulation process. On the contrary parametric approaches relay on strong assumptions about the theoretical properties of the risk factors which stand in forcing the later to comply to preset rules such as to follow a theoretical distribution. The Regulation focusses on the backtesting and the calibration and reviewing the risk model and sets some minimum standards to the performance of the models should meet.

Another term, quoted often, is the “volatility”; a sign that the Regulation recognizes that risk factors are heteroskedastic and that there is a need to go beyond static or unconditional models. It is unlikely that historical simulation (HS) would be acquiesced by the Regulation because the latter relies on unconditional or static volatility in generating scenarios. Another statistical term often used is “correlation” which is used in an interchangeable manner with the term “equivalent parameter of dependence”. Undoubtedly the Regulation recognizes that risk can be diversified away, and is open to the use of a statistical method for capturing the dependency between the risk factors. The technical term that the Regulation refers to more often, a total of thirty four times, is the “scenario” and “scenarios”. Scenario is synonymous with various types of simulation. Risk predictions are based upon hypothetical scenarios which are created to imitate the pathways the risk factors will follow. These scenarios are the product of the operation of a system over time. The system usually is constructed by a set of equations which replicate the behavior of the risk factors. The Regulation does not specify the type of the system or model to be developed in order to acquire the information about the behavior of the risk
factors. It is rather open to the system that generates scenarios. In order to meet the stress and backtests set by the Regulation, it is of paramount importance to build a simulation system which generates the scenarios that best match the behavior of these risk factors in the real world.

Market risk of swaps books is another example where the filtered historical simulation has been tested with success. It is essentially the fact that the value of a swap jumps immediately after a large payment is made, e.g. from +1 million to -2 millions when you receive 3 millions. The filtered historical simulation considers each payment date separately. The alternative practice of 'smoothing', i.e. taking the change in value from the daily changes in usual days, can be considerably inaccurate.

Appendix II

The ARMA-GARCH model, used by the filtered historical simulation, estimates conditionally the mean and variance in a joint system of equations as

\[
Y_t = \mu Y_{t-1} + \theta \varepsilon_{t-1} + \varepsilon_t \quad \varepsilon_t \mid I_{t-1} \sim N(0,h_t) \quad (1.a)
\]

\[
h_t = \omega + \alpha (\gamma + \varepsilon_{t-1})^2 + \beta h_{t-1} \quad (1.b)
\]

where \( \mu \) is the AR(1) term, \( \theta \) is the MA term and \( \varepsilon_t \) is the part of the daily price change that cannot be predicted by the ARMA terms and so remains a random variable. In the above parameterisation the conditional variance of \( Y_t \) is expressed as a (non-linear) function of past information. It validates earlier concerns about heteroskedastic stock returns, e.g. Mandelbrot (1963), Fama (1965), and meets the need for modelling the volatility as conditional on past information. Today’s variance depends upon yesterday’s unpredicted price change, \( \varepsilon_{t-1} \), and variance, \( h_{t-1} \). The terms \( \alpha \) and \( \beta \) do not need to sum to unity and one parameter is not a complement of the other. Their estimation is achieved by maximising the likelihood function. Thus the values of \( \alpha \) and \( \beta \) are critical in determining the current levels of volatility. Incorrect selection of the parameter values will adversely affect the estimation of volatility.

Today some CCPs model the risk factor’s volatility as time varying using a very restrictive version of the model in (1), the exponentially weighted moving average (EWMA) which is better known as exponential smoothing (ES). In the 1990s the ES
was widely used by the banks to estimate the volatilities and correlations in their daily VaR, see Riskmetrics\textsuperscript{15} (1995).

The EWMA restricts the $\mu$ and $\theta$ in (1.a) to be zero and specifies the conditional variance as:

$$h_t = \lambda h_{t-1} + (1-\lambda)Y_{t-1}^2 \quad 0 < \lambda < 1$$  \hspace{1cm} (2)

From (2) it emerges that the current level of volatility, $h_t$, is a function of yesterday’s volatility and the square of yesterday’s returns, $Y_{t-1}$. The EWMA has the advantage that it uses only one parameter, $\lambda$, but this forces last period’s return and volatility to have a unit effect on current period’s volatility. Thus, a large shock will have longer lasting impact on volatility in the model (2) than in (1). The assumption that $\alpha$ and $\beta$ sum to unity is a strong one and presents a testable hypothesis, often rejected, rather than a necessary condition. Furthermore, the ES assumes that the conditional mean of $Y$ is zero and ignores any serial dependency that may be present in the returns of the risk factor. When the conditional mean is different from zero the EWMA in (2) overestimates the variance.

Moreover, the model in (1.b) has an additional parameter, $\omega$, that acts as a floor and prevents volatility from dropping below that level. In the extreme case when $\alpha$ and $\beta$ equal zero, volatility is constant and equals $\omega$. The value of $\omega$ is estimated together with $\alpha$ and $\beta$ using maximum likelihood estimation and the hypothesis $\omega = 0$ can be tested easily. The absence of the $\omega$ parameter in the ES model allows volatility, after a few quiet trading days, to drop to very low levels, see Giannopoulos and Eales (1996).

Another appealing feature of the model in (1) is that it allows a flexible parameterisation in the variance and mean equations. Among others, Black (1976) observed that volatility tends to be higher when prices are falling than when prices are rising. This is known an asymmetry propriety of volatility and is also known as the leverage effect. The parameter $\gamma$ in (1.b) is to capture the presence of these types of asymmetries, and the effects on the risk factor’s volatility. A negative value confirms that the average impact on volatility of daily losses is larger than that of daily gains.

In the appendix A are reported the parameter estimates of the model in (1) for some of the data used in the filtered historical simulation backtesting in the Barone-Adesi et al. (2002) study. The ARMA-GARCH in (1) was fitted to all the series that formed the points in the yield curve of each currency and covers the data for the two years prior to the beginning of the backtesting period, i.e. 1994-1995. The aggregate value of $\alpha$ and $\beta$ is far below unity, rejecting the hypothesis that the volatility could be modelled as a EWMA process. Even more, in some series $\alpha$ is very large, revealing

\textsuperscript{15} Riskmetrics (1995).
that last trading day’s return has much larger impact in next day’s volatility, i.e. the volatility behaviour of that risk factor has short memory.

Berkowitz and O’Brien (2002) examined the accuracy of VaR models and compared the VaR forecasts versus the actual profits and losses (P&L) of six of the largest US banks. They found that the breaks, i.e. losses exceeding the VaR forecasts, were clustering during the Russian default crisis of the August-October 1998. An EWMA model, by far the most popular market risk model at that time, with a $\lambda$ set to a typical value of about 0.96, could not capture any clustering breaks. It will be very slow to react to few but large and consecutive movements in the interest rates. An $\lambda$ of 0.96 implies that in the ARMA-GARCH model in (1.b), $\beta$ is set to 0.96 and $\alpha$ to 0.04. To bear in mind that $\alpha$ and $\beta$ determine the smoothness of the volatility of the risk factor. The smaller is $\alpha$ and larger is the $\beta$ the smoother is the volatility. For most of the risk factors that we report in the appendix A, the aggregate $\alpha+\beta$ is much smaller than one\(^{16}\), rejecting the EWMA specification. For all the interest rates in the near end of the yield curve, the $\alpha$ is quite large, revealing a short memory in the volatility.

An EWMA specification with such a large $\lambda$ could explain the consecutive daily breaks for the VaR forecast of these banks portfolios. As Christoffersen (2003, p 183) points out, the breaks in adjacent days could bring a bank to collapse since the bank’s own capital will not be sufficient to cover the losses. Most importantly if these adjacent daily breaks could occur simultaneously across banks, it increases the risk of a systemic banking crisis.

Furthermore, in most series in Appendix A there are strong ARMA terms in the conditional mean equation, which, if they are in the data, cause some bias in the portfolio-generated scenarios. The $\gamma$ coefficient, which measures the asymmetric effects of the conditional volatility, is in about half of the series very large. Most of these coefficients have positive signs, which reveal these interest rates are making larger step upwards than downwards.

The first day variance is constant and it is known at the end the last trading day. Barone-Adesi et al (1999) showed the first day forecast for the mean and variance equation in a filtered historical simulation is given by

$$h_{t+1} = \omega + \alpha(\gamma + \varepsilon_t)^2 + \beta h_t$$  \hspace{1cm} (3.a)\(^{16}\)

\(^{16}\) The ARMA-GARCH model was estimated using the method of likelihood-likelihood. Additional tests were carried out on the residuals and conditional volatility estimates to ensure that the volatility model is the fit for every risk factor.
\[ Y_{t+1} = \phi Y_t + \theta \epsilon_t + e^* \cdot \sqrt{h_{t+1}} \] (3.b)

Where \( e^* \) are randomly drawn filtered residual returns. On the first day/step of the simulation, the number of unique scenarios equals the number of observations in the historical data period. There is no need to draw random residuals to fit in (3.a). The simulated density of the portfolio profits and losses, in the first step ahead, is accurately constructed by fitting each of the residual returns in (3.a). The constant volatility in the first period ahead, \( \sqrt{h_{t+1}} \), rescales but does not alter the shape of the distribution of the set of filtered residual returns. Indeed the simulated density of \( Y_{t+1} \) will be a perfect match of distribution of \( e^* \) but rescaled by \( \sqrt{h_{t+1}} \).

By drawing a larger number random standardised returns, \( e^* \), than the size of the historical dataset, this will only generate replicated values for \( Y_{t+1} \). Unless the number of simulation trials is a large multiplier of the size of the historical dataset, then the empirical distribution of \( Y_{t+1} \) will mismatch the one of the \( e^* \) original series because of sampling errors. Attention is needed when the risk exposure is measured by the fractiles of these simulated portfolio values. In the case where the number of simulation trials is larger than the observations in the historical dataset, there will be a repetition of the scenarios. For example, running a quasi filtered historical simulation on 5,000 trials and two years of historical returns, each unique scenario will occur on average 10 times. But due to sampling error the 10 to 1 rule will be violated.

On the longer horizon the filtered residuals are drawn with replacement. On each risk factor the most extreme scenario is the one formed by the consecutive drawings of the largest negative/positive filtered return in the historical dataset. What combination of scenarios cause the absolute extreme loss in a portfolio depends on the portfolio positions.

The forecast for the quasi filtered historical simulation is given by

\[ Y_{t+1} = \frac{Y_t - h_t}{\sqrt{h_t}} \cdot \sqrt{h_{t+1}} \] (4.a)

Where \( h_{t+1} \) is given by

\[ h_{t+1} = \lambda h_t + (1-\lambda) Y_t^2 \] (4.b)
By replacing in (4.b) each of the parallel filtered residuals returns, all feasible scenarios are exhausted. Any new scenarios can only be obtained by extending back in time the historical period. The distribution of the generated scenarios, for $Y_{t+1}$ and so for the risk factor prices, reflects the empirical distribution of the historical data set of filtered residual returns.

The quasi filtered historical simulation fails to account for changing volatility, because it estimates longer period risk exposure as a multiple of that first period quantile and the square root of the time.